Working With GABE

Some Practice with Numerical Methods in Preheating

Some relations that are important, and probably could use a reminder are

$$H^2 = \frac{8\pi}{3m_{\rm pl}^2}\rho$$

where

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{(\nabla\phi)^2}{2a^2} + V(\phi).$$

Challenge 1. We'd like to explore *tachyonic* preheating, for the case where we have to do minimal modifications to the `g2model.cpp` file. Consider the following potential,

$$V(phi) = \frac{1}{2}m^2\phi^2 + \sigma\phi\chi^2 + \frac{\lambda_\chi}{4}\chi^4 \tag{1}$$

although we'll eventually set $g^2 = 0$, I would recommend not removing that term from the code.

(a) Keeping the rescaling, $B = m$, what are the new dimensionless couplings you need. Remember that

$$V_{\rm pr} = \frac{V}{B^2 m_{\rm pl}^2}.$$

(b) Now you have to do several (small) modifications to the code. Starting in `g2parameters.h`, let's add the two, physical, values of our new parameters. Somewhere around line 46, you'll want to add the two newlines. I'm going to call my parameters `sig`, which is going to be the value $\sigma/m_{\rm pl}$ and `lambdachi`, which is going to be the (already dimensionless, λ_χ. Let's also set $g^2 = 0$, while we're here.

I tend to keep these in Planck units in the `g2paramer.h` file, but that's up to you. For these simulations we're going to be inspired by `https://arxiv.org/pdf/hep-ph/0602144` and choose

$$\sigma = 5 \times 10^{-10}$$

and

$$\lambda_\chi = 2.5 \times 10^{-7}.$$

(c) Now let's focus on `g2model.cpp`, define your two new dimensonless couplings (in our around line 64), then add the needed terms to the three functions.

(d) The mode equations for the coupled field are

$$\ddot{\chi}_k + 3H\dot{\chi} + \left(\frac{k^2}{a^2} + 2\sigma\langle\phi\rangle\right)\chi_k = 0 \tag{2}$$

Since the χ field will start with zero mean. Therefore the effective mass,

$$m_{\rm eff}^2 = \frac{k^2}{a^2} + 2\sigma\langle\phi\rangle$$

will be negative for some values of σ, when $\phi < 0$. This will be strongest when the field has its maximum negative value. At this point (in the homogeneous model)

$$\phi_{\rm min} \approx -0.05\, m_{\rm pl}$$

when

$$a \approx 2.$$

What is k_*, the largest value of k that is tachyonic at this point?

(e) We will still start the simulation at the end of inflation, as we did before with

$$\phi_0 = 0.193\, m_{\rm pl}$$

and

$$\dot{\phi}_0 = -0.1422\, m\, m_{\rm pl}.$$

What is the smallest wave-number of a dynamical mode, assuming that ($k_{\rm min} = H$) at the end of inflation? Please remember that we use the regular Planck mass, $m_{\rm pl} = G^{-1/2}$. This value of $k_{\rm min}$ sets the lower-end of the tachyonic instability. What is a good value of L to resolve this mode (remember that $\Delta k = 2\pi/L$).

(f) Let's make sure that we can resolve the tachyonic instability. Given that the largest k on the lattice is

$$k_{\rm nyquist} = \frac{\sqrt{3}}{2} N \times \Delta k = \frac{\sqrt{3}}{2} \times 64 \times \frac{2\pi}{L}$$

Is $k_{\rm nyquist}$ sufficiently larger than k_*?

(g) Run the simulation and check to see that preheating happens!

(h) Look at the spectra produced during the time when the ϕ-field is at it's minimum (around $t = 3\, m^{-1}$). Can you verify that the amplified modes lie between $k_{\rm min}$ and k_{rmmax}?

Challenge 2. Try to make some Oscillons! Note that these generally require resolution greater than $N = 64$, but it is a fun exercise. Consider the Axion Monodromy potential,

$$V(\phi) = m^2 M^2 \left(\sqrt{1 + \frac{\phi^2}{M^2}} - 1 \right) \tag{3}$$

where we have two free parameters, yet only one field.

(a) Start by writing this potential in dimensionless units. Please remember that we can't change the scaling of the fields, $\phi_{\rm pr} = \phi/m_{\rm pl}$!

(b) Now calculate the other two functions we need in order to modify `g2model.cpp`, namely,

$$\frac{\partial V_{\rm pr}}{\partial \phi_{\rm pr}}$$

and

$$m_{\rm eff}^2 = \frac{\partial^2 V_{\rm pr}}{\partial \phi_{\rm pr}^2}.$$

(c) When you're ready code these into `g2model.cpp`.

(d) The final thing we need are initial conditions. This can be done using the homogeneous field limit (during inflation.) When we do this we get

$$\phi_0 = 0.2013\, m_{\rm pl}$$

$$\dot{\phi}_0 = -0.1006\, m\, m_{\rm pl}$$

when $m = 2.2 \times 10^{-5} m_{\rm pl}$ and $M = 4000\, m_{\rm pl}$.

(e) Calculate what H^{-1} and set the box size to be few $\times H$.

(f) Plot the variance of the ϕ-field to look for indications of resonance and, hence, indications that the field has fragmented.

(g) Now, make sure that you have field slices turned on, and 3-dimensional. See if you can plot a slice after you think oscillons have formed. Can you find them? Be sure to plot ρ!

(h) Finally, we expect the simulation to stop making sense when the oscillons (which have width $\sim m^{-1}$) approach the lattice spacing, Δx. Can you calculate when this happens? How long can you trust your simulations?